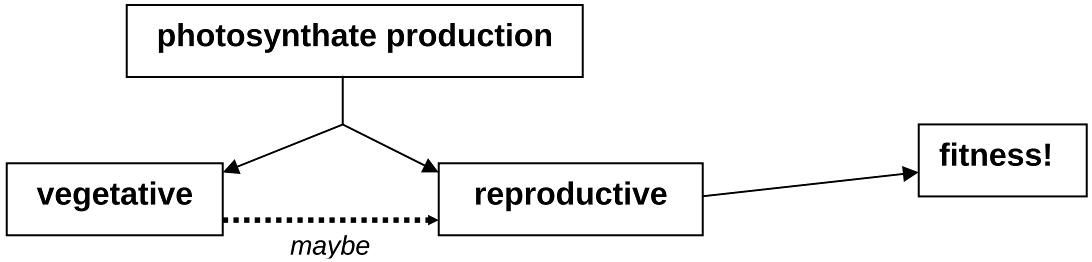
Annual plant

- Two compartments: reproductive and vegetative:



- Reproductive size is what counts, but allocation to vegetative growth could increase long-term reproductive size.

Vegetative size

- First day's vegetative size is v_1.
- After first day, vegetative size is previous vegetative size, plus vegetative increment: $v_t = v_{t-1} + x_{t-1}$.
- Vegetative increment is proportional to vegetative size and to fraction allocated to vegetative growth: $x_{t-1} = (1-b_{t-1})L$.
- So, second day's vegetative size is: $v_2 = v_1 + v_1(1-b_1)L = v_1(1+(1-b_1)L)$.
- Third day:

$$v_3 = v_2(1+(1-b_2)L)$$
$$= v_1(1+(1-b_1)L)(1+(1-b_2)L)$$

- Notice that each subsequent day *t*, we're just going to tack an extra $(1+(1-b_t)L)$ to the end of a bunch of other $(1+(1-b_?)L)$'s. For example:

$$v_{10} = v_1(1+(1-b_1)L)(1+(1-b_2)L)(1+(1-b_3)L)(1+(1-b_4)L)(1+(1-b_5)L)(1+(1-b_6)L)(1+(1-b_7)L)(1+(1-b_8)L)(1+(1-b_9)L).$$

- For ease of notation, we'll just write this:

$$v_{10} = v_1 \prod_{u=1}^{9} (1+(1-b_u)L).$$

- Or, in general: $v_t = v_1 \prod_{u=1}^{t-1} (1 + (1 - b_u)L)$.

Reproductive size

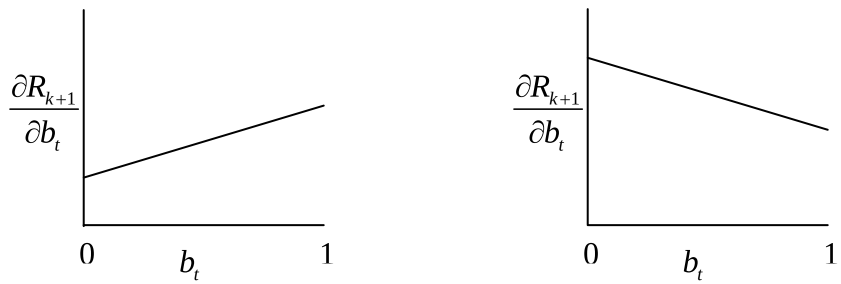
- First day's reproductive size is 0.
- After first day, reproductive size is previous day's reproductive size, plus reproductive increment: $R_t = R_{t-1} + r_{t-1}$.
- Reproductive increment is proportional to vegetative size and to fraction allocated to reproductive growth: $r_{t-1} = b_{t-1} A v_{t-1}$.
- For the final reproductive size (on day *k*), just sum up all the increments:

$$R_{k+1} = \sum_{u=1}^{k} b_u A v_u$$

$$= \sum_{u=1}^{k} \left[b_u A v_1 \prod_{u=1}^{t-1} (1 + (1 - b_u)L) \right]$$

What to do with all that

- It'd be nice to find a sequence of *b*'s that maximizes final reproductive size. Then we could go out and see whether real plants have optimal allocation schedules.
- Unfortunately, the math to do that is too hard. So, let's look at an easier problem: if you know the *b*'s for every day except for one, what *b* for that day maximizes final reproductive size?
- Differentiate the fecundity function (i.e., R_{k+1}, above) with respect to b_t, and find what b_t gives a zero derivative.
- The derivative function turns out to be a straight line. The line doesn't intersect the b_t-axis anywhere in $0 \le b_t \le 1$. The slope is positive when *t* is small, negative when *t* is large.



- Thus the bang-bang allocation schedule.

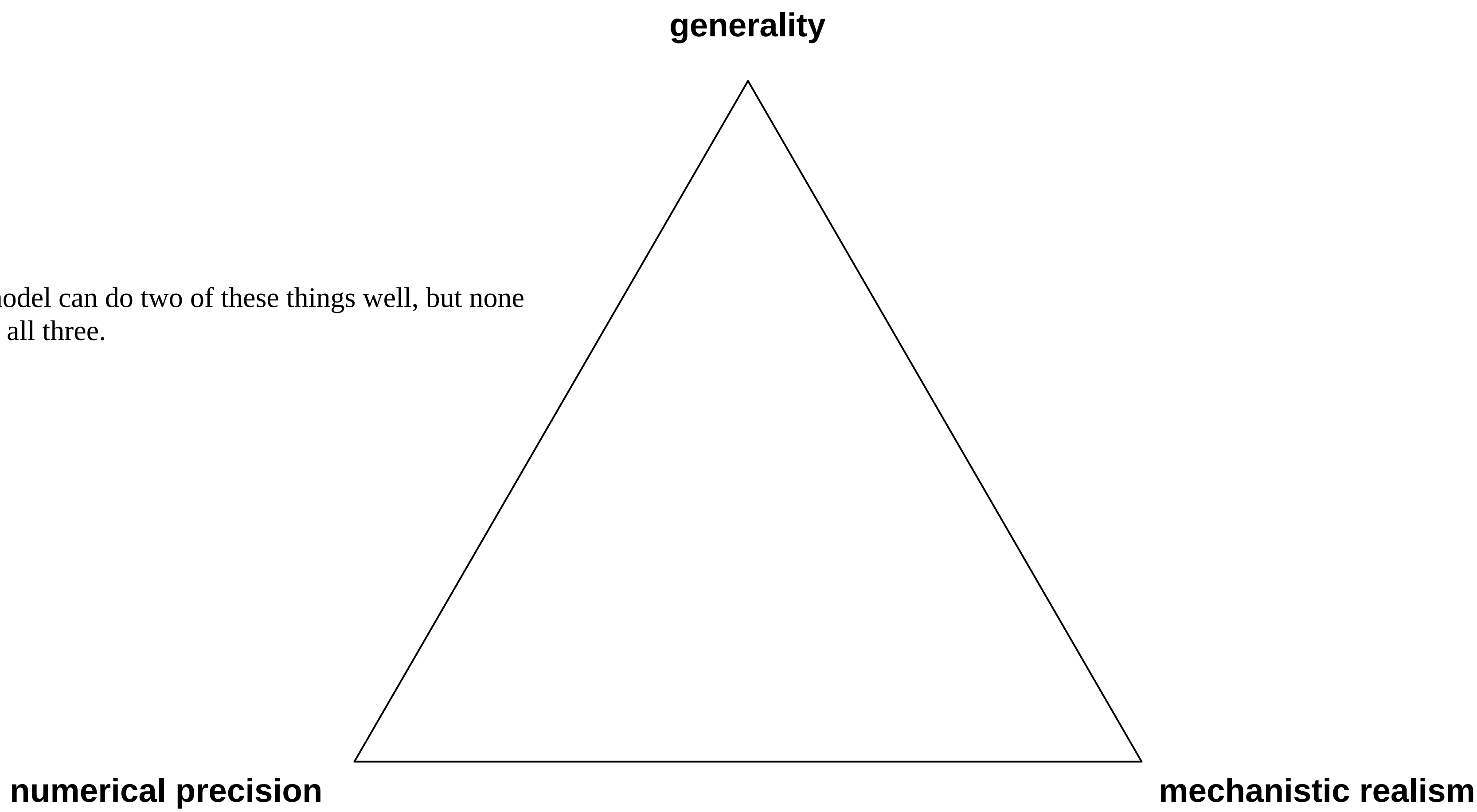
What do we think about that?

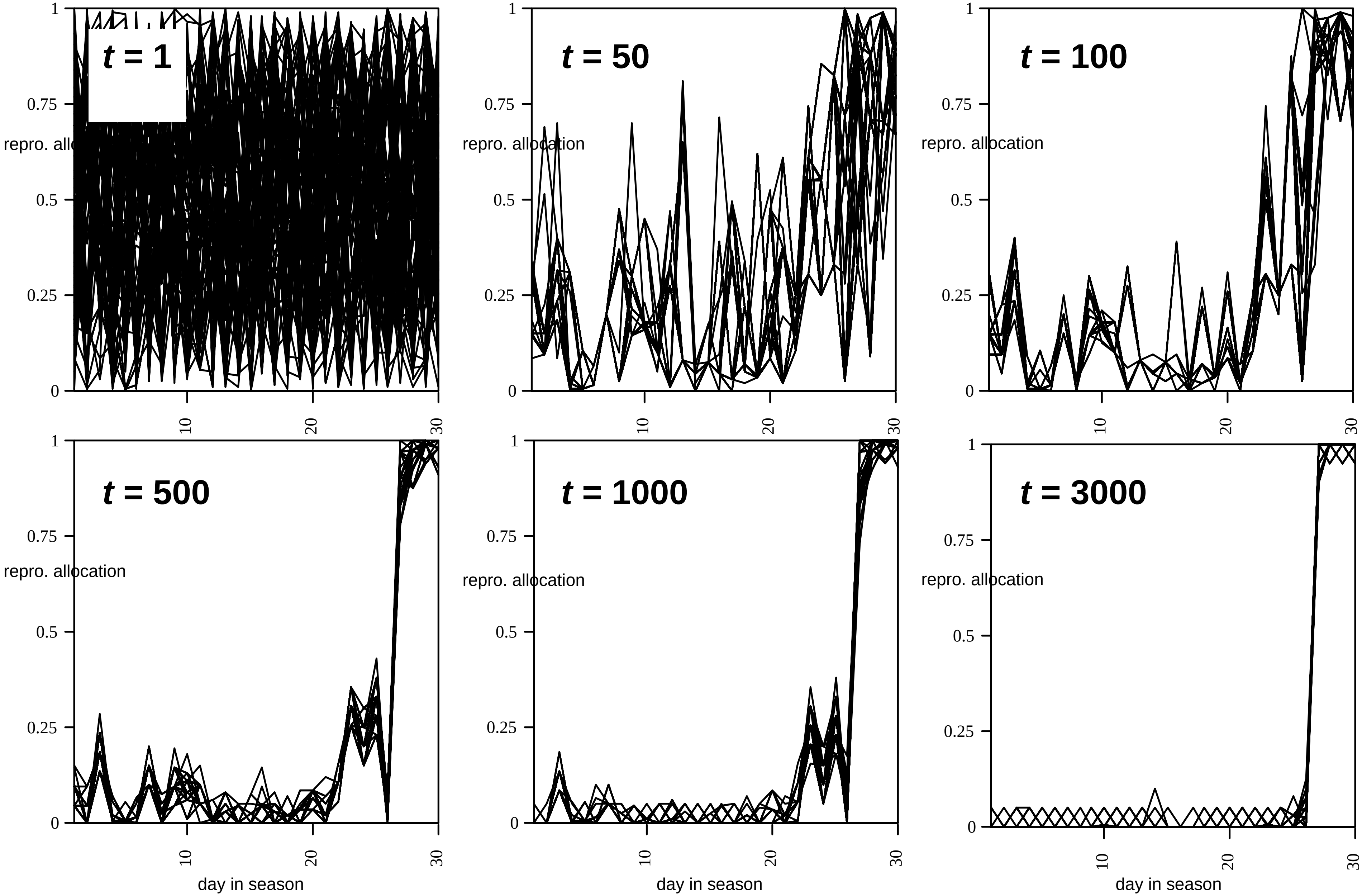
- How general is the model? What range of real-life situations does it cover?
- What silly assumptions does it make?
- How might we improve it?
- Do these improvements change the generality of the model?

What is a model, anyway?

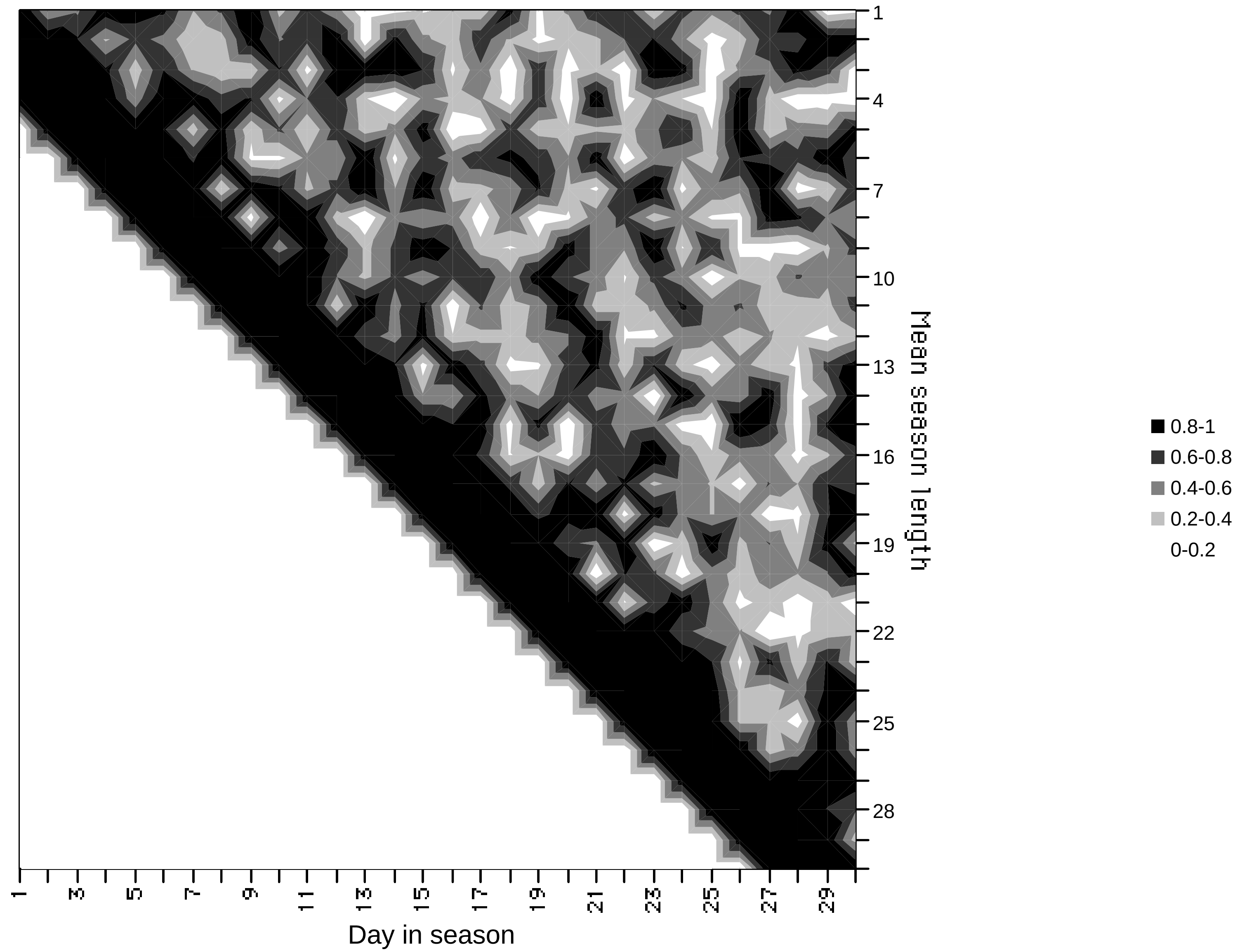
- We claim to be interested in plants, but all we're doing is manipulating equations.
- We left out all sorts of things, and completely fabricated others.
- That's science? Changing the subject and making things up??

Any model can do two of these things well, but none can do all three.

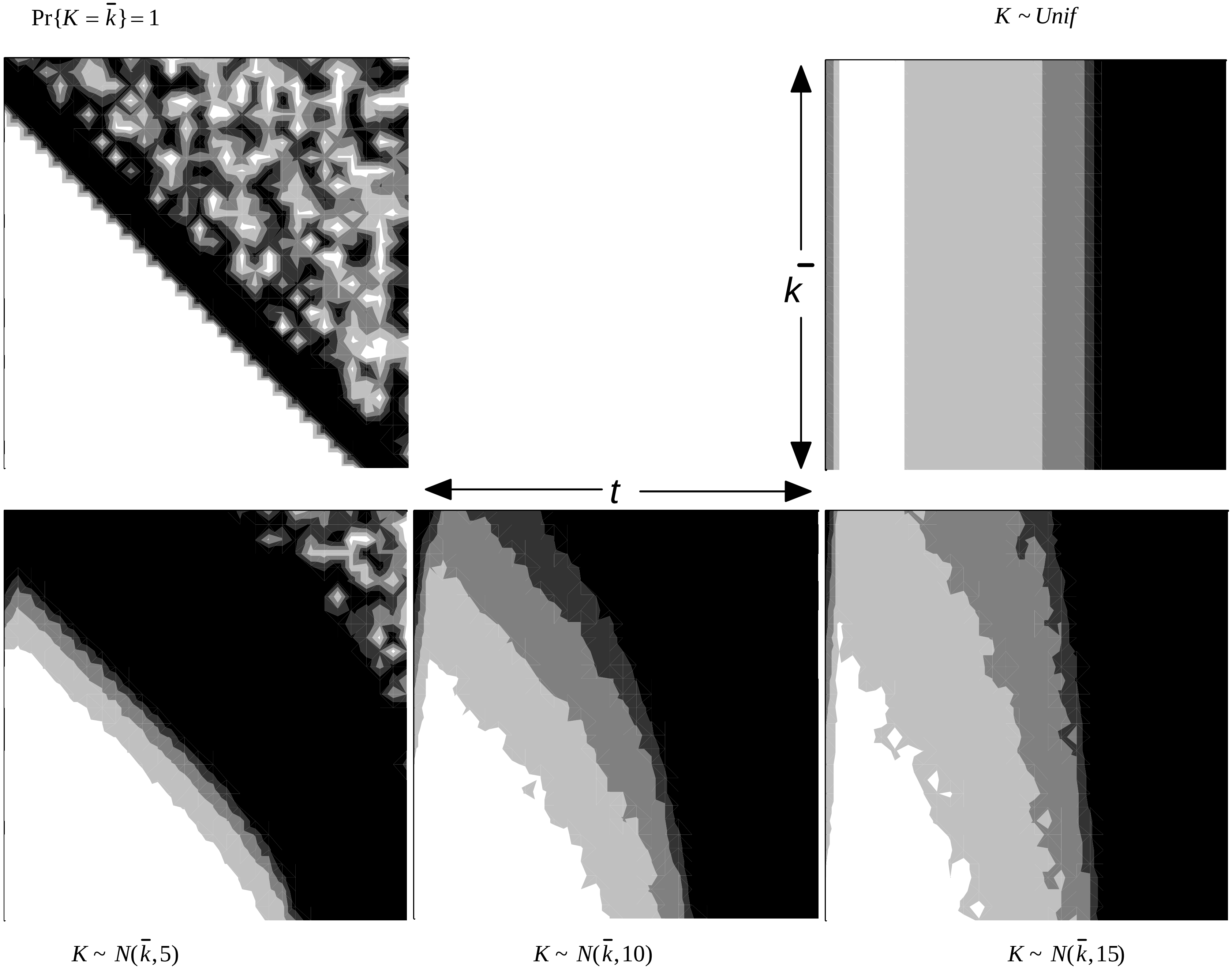


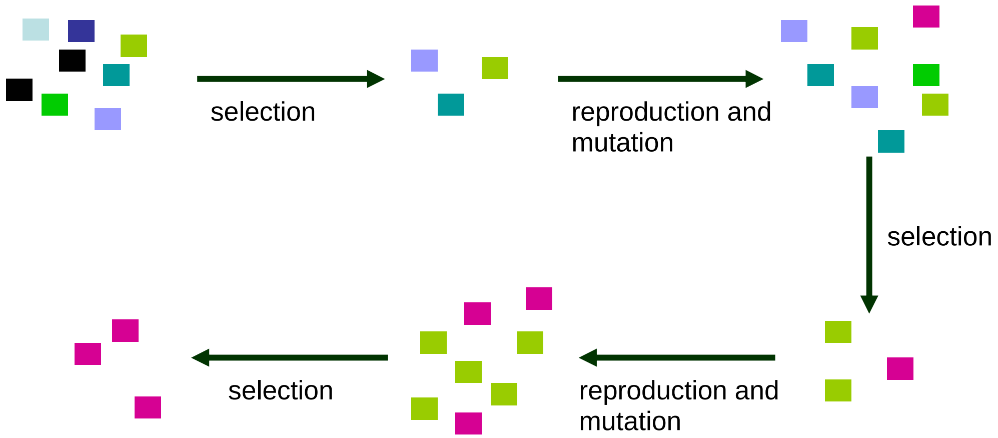


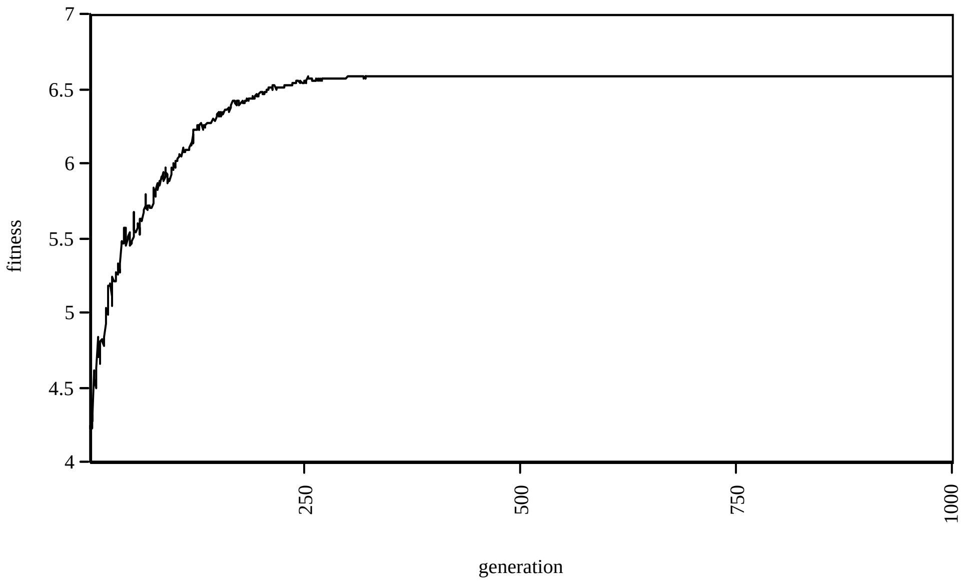
t = 1
t = 50
t = 100
t = 500
t = 1000
t = 3000
repro. allocation
day in season
0
0.25
0.5
0.75
1
10
20
30



Mean season length
1
4
7
10
13
16
19
22
25
28
Day in season
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
0.8-1
0.6-0.8
0.4-0.6
0.2-0.4
0-0.2


$\Pr\{K = \bar{k}\} = 1$
$K \sim Unif$
$\bar{k}$
t
$K \sim N(\bar{k}, 5)$
$K \sim N(\bar{k}, 10)$
$K \sim N(\bar{k}, 15)$


selection
reproduction and mutation
selection
reproduction and mutation
selection


7
6.5
6
5.5
5
4.5
4
fitness
250
500
750
1000
generation