<u>Vector Fields:</u>

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<u>GROUP MEMBERS:</u>

1. ____________________________

2. _____________________________

3. _____________________________

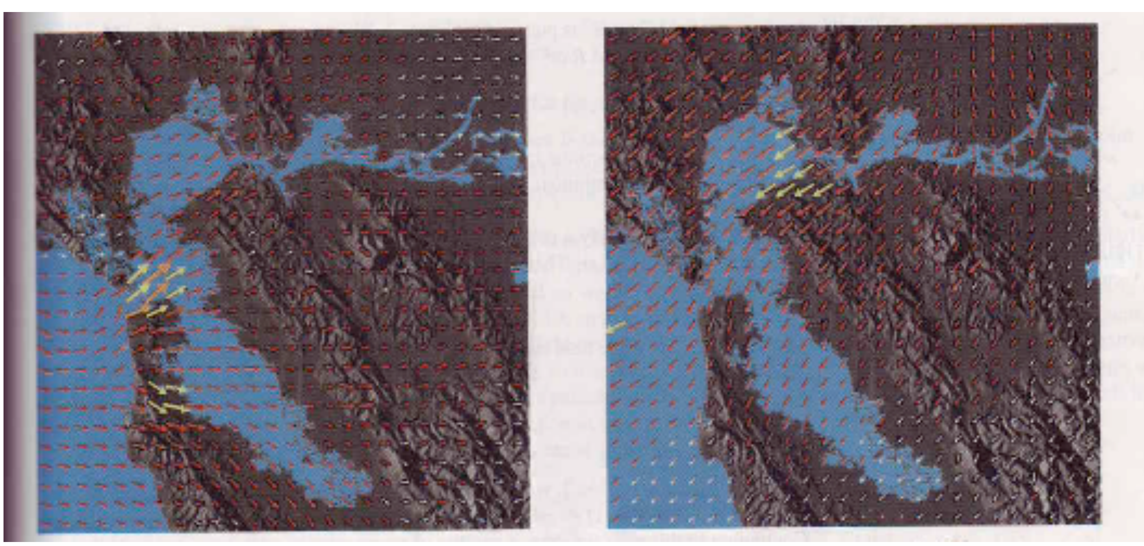
4. _____________________________

5.

<u>Goal:</u> Learn how vector fields can be used to visually convey information and how one can mathematically define and graph vector fields.

Part I: **<u>Exploration</u>:**

1. What do the following two pictures show? What is the same about the pictures? What is different? What factors might account for the differences?



a. 5:00 pm August 6, 1997 b. 1:00 pm January 7, 1998

Vectors showing wind patterns in San Francisco Bay

(Taken from Multivariable Calculus, 4th Edition, Stewart, Brooks/Cole Publishing).

2. Find other examples where a collection of vectors as above conveys information. Describe what the vectors are showing.

3. A collection of vectors as above is called a ***vector field***. Why do you think this name is used?

Part II: **The Mathematics of Vector Fields.**

Directions: One person works at a time. Person 1 does first problem**. Explain to the group what you are doing**. Write out your work and answer on the sheet. When done, ask whether everyone in the group understands. People in the group ask questions. Pass the sheet to person 2 who will follow the same procedure and do second problem. Then person 3 does third problem and so on.

Calculate the values of the function

$$F(x,y) = (-x - y, x^2)$$

at the following points. ***After all four of these values have been calculated***, then plot these vector on the next sheet .

Demo Example: $F(x = 1, y = 1) = (-1 -1, 1^2) = (-2, 1)$

At the point (1, 1) we plot the vector (-2, 1). Think of drawing a mini set of x-y axes centered at the point (1, 1). (see plot on next page).

Now you do it. First calculate the vectors. After everyone has had their turn to calculate, then each person will plot their vector at the appropriate point (next page).

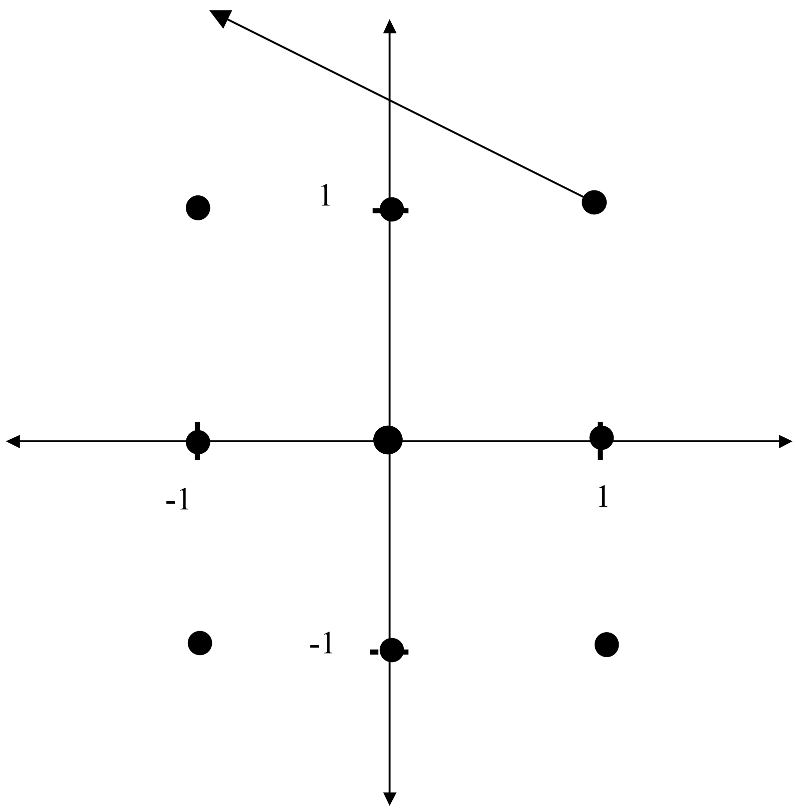
Person 1. F(1, 0) =

Person 2. F(0, 1) =

Person 3. F(1, -1) =

Person 4. F(-1, 1) =

Now graph these vectors. Continue in the same order. First find the point (x, y) on the graph and label the point. Then starting at that point, plot the vector.



Continue and calculate these values using the same formula

$$F(x,y) = (-x - y, x^2)$$

Person 1. F(0, 0) =
Person 2. F(-1, 0) =
Person 3. F(-1, -1) =
Person 4. F(0, -1) =

Now plot these points and vectors on the graph.

Part III: **Summary:**

4. When your group was learning how to plot vectors in a vector field, what (if any) confusions arose?

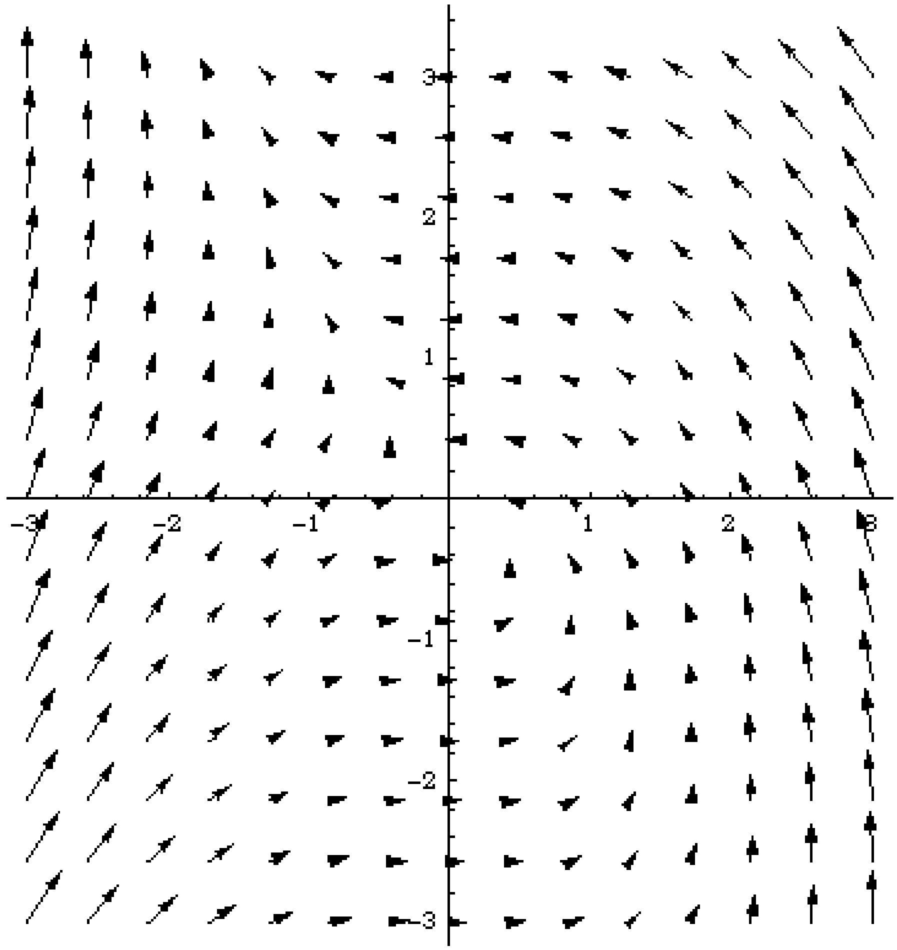
5. Imagine you are describing what you learned today to a classmate who was absent from class. Explain the process graphing vectors in a vector field.

Definition: The function $F(x,y) = (-x - y, x^2)$ is an example of what is called a **vector valued function** since for each value of (x,y) the output of the function is a vector. We also call the function F(x,y) a **vector field.**

Technology and Vector Fields: After a while, it becomes boring to calculate and plot a vector field by hand. Fortunately, graphing calculators and computers can do these calculations quickly and accurately. Below are commands from the computer algebra system Mathematica to plot $F(x,y) = (-x - y, x^2)$.

In[1]:= << Graphics`

In[2]:= PlotVectorField[{-x - y, x^2}, {x, -3, 3}, {y, -3, 3}, Axes → True]



Note that the length of the vectors in this picture are not drawn to scale. Mathematica is automatically adjusting the lengths so that the vectors doe not overlap. However the directions of the vectors are still correct.

Imagine that the vector field is indicating the flow patter for water in a river. When you drop a small piece of wood in the river, the arrows show the way the wood will be pushed along by the current.

Each person take a turn placing a dot somewhere on the bottom of the above figure. This is where you are dropping your piece of wood. Now draw the path that the wood would follow from that starting point. The paths you are drawing are called **flow lines of the vector field.**

What do you observe about the overall pattern?